Comment on "Dynamic properties in a family of competitive growing models"

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The article [Phys. Rev. E **73**, 031111 (2006)] by Horowitz and Albano reports on simulations of competitive surface-growth models RD+X that combine random deposition (RD) with another deposition X that occurs with probability p. The claim is made that at saturation the surface width $w(p)$ obeys a power-law scaling $w(p) \propto 1/p^{\delta}$, where δ is only either $\delta=1/2$ or $\delta=1$, which is illustrated by the models where X is ballistic deposition and where X is RD with surface relaxation. Another claim is that in the limit $p \to 0^+$, for any lattice size L, the time evolution of $w(t)$ generally obeys the scaling $w(p,t) \propto (L^{\alpha}/p^{\delta}) F(p^{2\delta} t/L^{z})$, where F is Family-Vicsek universal scaling function. We show that these claims are incorrect.

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In Ref. [1] the following scaling ansatz is proposed:

$$w^2(p,t) \propto \frac{L^{2\alpha}}{p^{2\delta}} F\left(p^{2\delta}\frac{t}{L^z}\right), \tag{1}$$

where $w(p,t)$ are time evolutions of surface width in competitive growth models RD+X when a random deposition (RD) process is combined with process X, and $p \in (0;1]$ is the selection probability of process X. The function $F(\cdot)$ represents Family-Vicsek universal scaling. The ansatz (1) has been studied previously [2–4] by examples where X represented either Kardar-Parisi-Zhang or Edwards-Wilkinson universal process. The new claim that is being made in Ref. [1] is that a nonuniversal and *model-dependent* exponent δ in Eq. (1) must be only of two values, either $\delta=1$ or $\delta=1/2$, for models studied in Ref. [1]. To show that this claim is not correct we performed (1+1) dimensional simulations of RD+X models when X is ballistic deposition (BD) and when X is random deposition with surface relaxation (RDSR), and performed scaling in accordance to Ref. [1]. Our results are presented in Figs. 1–3.

Our data have been obtained on L site lattices (L is indicated in the figures) with periodic condition, starting from initially flat substrates, and averaged over 400 to 600 independent configurations. The time t is measured in terms of the deposited monolayers. Simulations have been carried up to $t=10^7$, and the surface width at saturation has been averaged over the last 5000 time steps. The data sets are for ten equally spaced selection probabilities p from $p=0.1$ to $p=1$, where $p=0$ would be for RD process with no X present, and $p=1$ is for process X in the absence of RD. The data have been scaled in L with the theoretical values of universal roughness exponent α and dynamic exponent z of the universality class of process X. The RDSR algorithm used in our simulations is given in Ref. [5] (Sec. 5.1). The BD algorithm used as X=BD1 is the nearest-neighbor (NN) sticking rule found in Ref. [5] (Sec. 2.2), and the BD algorithm used as X=BD2 is the next-nearest-neighbor (NNN) sticking rule found in Ref. [5] (Sec. 8.1).

Saturation. Saturation data (Fig. 1) show that in special cases an approximate power law $w(p) \propto 1/p^{\delta}$ may be observed. However, this is not a principle. Even if the data can be fit to the power law in p only one of our examples shows a reasonable fit with $\delta \approx 1$ [seen in Fig. 1(a)]. When X =BD1 the data in Fig. 1(b) show $\delta < 1/2$. The other two examples shown in Fig. 1 defy a linear fit. In these cases there is no power law of the type claimed in Ref. [1]. This absence of power-law scaling in p is also evident in Fig. 4 of Ref. [1].

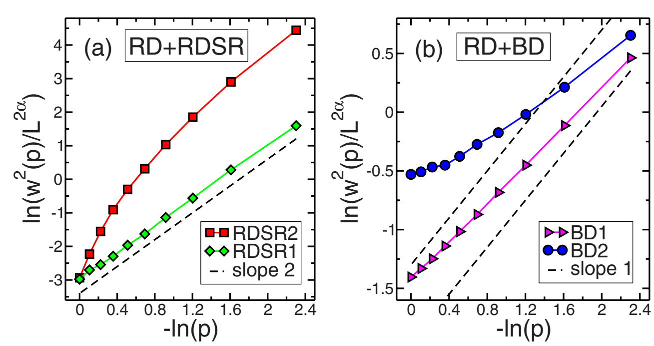

FIG. 1. (Color online) Interface width at saturation in the RD +X model vs the selection probability p of process X. (a) X is RDSR: the case when both RD and RDSR deposits are of unit height (diamonds, RDSR1; $L=500$); and, the case when RDSR deposits are of unit height and RD deposits are of twice that height (squares, RDSR2; $L=100$). (b) X is BD: the case of the NNN rule (circles, BD2); and, the case of the NN rule (triangles, BD1). In RD+BD simulations $L=500$. Solid line segments connecting data points (symbols) are guides for the eye. The dashed lines give reference slopes.

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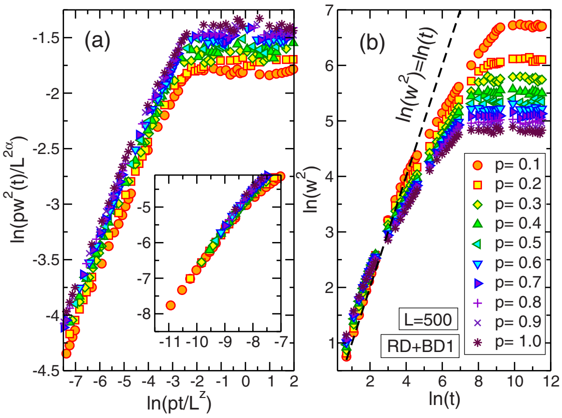

FIG. 2. (Color online) Time evolutions $w^2(p,t)$ in RD+BD1. (a) Scaling in p after Ref. [1]. The inset shows the scaled initial transients. (b) Evolution curves before scaling. The dashed line is the RD evolution for $p=0$. In all models when the simulations start from a flat substrate $w(t)$ must pass an initial transient before universal scaling can be measured. The initial transients in part (b) follow RD universal evolution.

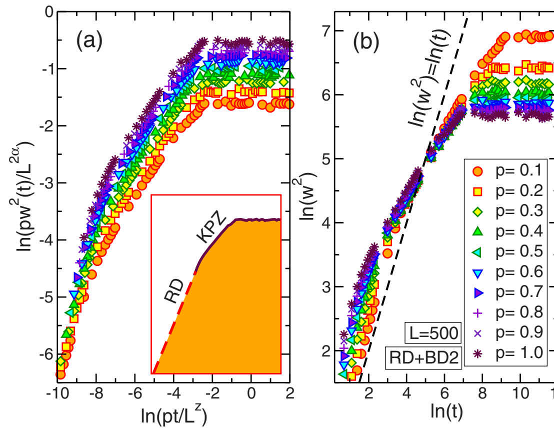

FIG. 3. (Color online) Time evolutions $w^2(p,t)$ in RD+BD2. (a) Scaling in p after Ref. [1]. The outcome of this scaling is summarized in the inset. (b) Data before scaling. The dashed line is the RD evolution for $p=0$.

RD limit. Another claim of Ref. [1] is that Eq. (1) with the power-law prefactors p^δ (where $\delta=1$ or 1/2) would prevail in the RD limit of $p\to 0^+$, and that such a scaling would be universal. We tested these claims in simulations of RD+BD models and found the evidence to the contrary (Figs. 2 and 3). In order to prove the absence of power-law scaling via Eq. (1) in the RD limit we present in Figs. 2(b), 3(a), and 3(b) the original $w^2(p,t)$ data before scaling. These original data show that parameter p, $p\in(0;1]$, assigns an order in the set of all curves $w^2(p,t)$ in such a way that $w^2(1,t)$ is the lowest-lying curve, and at $p=0$ the initial transients become the RD universal evolution $w^2_{\mathrm{RD}}(0,t)\propto t$. The region between the boundaries $w^2(1,t)$ and $w^2_{\mathrm{RD}}(0,t)$ is densely covered by the curves $w^2(p,t)$ because p takes on continuous values. The pattern shown in Figs. 2(b), 3(a), and 3(b) for $p\in[0.1;1]$ extends down to values that are infinitesimally close to $p=0$, i.e., to the entire range of p. If the simulations are stopped at infinitesimally small p' the width $w^2(p',t)$ is always the highest lying curve in Figs. 2(b), 3(a), and 3(b). In other words, the smaller the p' the higher the saturation value of $w^2(p',t)$. But there is no bounding highest curve $w^2(p',t)$ in this set since the boundary $w^2(0,t)$ is the RD evolution. This order is reversed under the scaling of Eq. (1) when we set $\delta=1/2$, following Ref. [1]. The outcome of this scaling is seen in Figs. 2(a), 2(b), and 3(a): the boundary $w^2(1,t)$, i.e., the lowest-lying curve in Figs. 2(b), 3(a), and 3(b), is mapped onto the highest-lying curve in the image of this scaling seen in Figs. 2(a), 2(b), 3(a), and 3(b); and, a higher-lying curve $w^2(p,t)$ before scaling in Figs. 2(b), 3(a), and 3(b) is mapped onto a lower-lying curve after scaling in Figs. 2(a), 2(b), and 3(a). In this scaling, the initial transients become ever longer as p becomes ever smaller and closer to $p=0$, as seen in the inset in Fig. 2(a). For any range of p, also in the limit $p\to 0^+$, the image of this scaling demonstrates no data collapse. This image is shown in the inset in Fig. 3(a). Hence, for RD+BD models Eq. (1) with $\delta=1/2$ does not produce data collapse.

In some instances of model X, however, Eq. (1) can give an *approximate* data collapse [3,4] but then δ is not restricted to the two values postulated in Ref. [1]. For example, for the RD+BD1 model such scaling can be obtained with $\delta\approx 0.41$ [3] [note, $0.4<\delta<0.5$ is seen in Fig. 1(b)]. But for the RD+BD2 model there is no value of δ that would produce data collapse when nonuniversal prefactors in Eq. (1) are expressed as a power law p^δ. We have demonstrated that such scaling does not generally exist and if occasionally it is observed it is a property of particular model.

In summary, the form of the nonuniversal prefactors as seen in universal Eq. (1) is a fit and is *not* a principle. The exponent δ in Eq. (1) is model dependent, and the prefactor that enters may have other forms than p^δ.

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